Chapter 2, FORECASTING

Companies use demand forecasting for:

- Production Planning
- Inventory Management
- Capacity Planning, etc.

Forecasting Methods:

Subjective
- Sales Force Composites
- Customer Surveys
- Jury of Executive Opinion
- The Delphi Method

Objective
- Causal Models
- Time Series

Question 1 What are the advantages and disadvantages of objective vs. subjective forecasting methods?
Causal Models:

- Data other than the series predicted used.
- $Y$: Dependent Variable,
  $X_1, X_2, \ldots, X_n$: Independent Variables
- In general, $Y = f(X_1, X_2, \ldots, X_n)$.
  A special case, $Y = \alpha_0 + \alpha_1 X_1 + \ldots + \alpha_n X_n$.

Example 1 Campus Security Department is trying to estimate the number of security people to be appointed during the next semester. The policy is to choose this number in accordance with the estimated number of criminal events. Research has shown that there exists a close relationship between the violent movies shown in town in the current semester and the number of criminal events committed in the following semester. The predicted relationship is:

$$Y_{i+1} = \alpha_0 + \alpha_1 X_i,$$

and the past data is

<table>
<thead>
<tr>
<th>$i$</th>
<th>$x_i$</th>
<th>$y_{i+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>26</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>18</td>
</tr>
</tbody>
</table>
Example 1, Cont’d

Given that \( X_5 = 7 \), find an estimate for the number of crimes in the 6th semester (i.e., \( \hat{Y}_6 \)).

Solution

- Find the least squares estimators for \( \alpha_0 \) and \( \alpha_1 \). That is, find the values of \( \alpha_0 \) & \( \alpha_1 \) such that, the sum of the squares of the distances from the line \( \alpha_0 + \alpha_1 X_{i-1} \) to the observed number of crimes, i.e. \( Y_i \), is minimized.

\[
d(\alpha_0, \alpha_1) = \sum_{i=1}^{4} \left[ y_{i+1} - (\alpha_0 + \alpha_1 x_i) \right]^2.
\]

In this specific example,

\[
d(\alpha_0, \alpha_1) = [7 - (\alpha_0 + 2\alpha_1)]^2 + [26 - (\alpha_0 + 8\alpha_1)]^2
\]
\[
+ [19 - (\alpha_0 + 6\alpha_1)]^2 + [18 - (\alpha_0 + 5\alpha_1)]^2
\]

The first derivatives are given by

\[
\frac{\partial d(\alpha_0, \alpha_1)}{\partial \alpha_0} = -140 + 8\alpha_0 + 42\alpha_1,
\]
\[
\frac{\partial d(\alpha_0, \alpha_1)}{\partial \alpha_1} = -852 + 42\alpha_0 + 258\alpha_1.
\]

To see if \( d(\alpha_0, \alpha_1) \) is jointly convex w.r.t. \( \alpha_0 \) and \( \alpha_1 \), check if the determinant of the Hessian Matrix is greater than zero.
\[
\frac{\partial^2 d(\alpha_0, \alpha_1)}{\partial \alpha_0^2} = 8, \quad \frac{\partial^2 d(\alpha_0, \alpha_1)}{\partial \alpha_1^2} = 258,
\]
\[
\frac{\partial^2 d(\alpha_0, \alpha_1)}{\partial \alpha_0 \partial \alpha_1} = \frac{\partial^2 d(\alpha_0, \alpha_1)}{\partial \alpha_1 \partial \alpha_0} = 42.
\]

Now, we can calculate the determinant of the Hessian Matrix.

\[
\det \begin{vmatrix} 8 & 42 \\ 42 & 258 \end{vmatrix} = 300 > 0
\]

Therefore, the following equations give the unique minimizer of \(d(\alpha_0, \alpha_1)\).

\[
\frac{\partial d(\alpha_0, \alpha_1)}{\partial \alpha_0} = 0 \quad \frac{\partial d(\alpha_0, \alpha_1)}{\partial \alpha_1} = 0.
\]

It turns out that \(\alpha_0 = 1.12\) and \(\alpha_1 = 3.12\). Hence, \(\hat{Y}_6 = 23\).\[\square\]
Time Series Models:

- Past observations of the series being predicted are used.

Time Series Patterns

- a. No pattern
- b. Trend (Increasing linear)
- c. Trend (Exponential)
- d. Seasonal

- For Stationary Series
  - Moving Averages
  - Exponential Smoothing

- Trend-Based Methods
  - Regression Analysis
  - Double Exponential Smoothing

- For Seasonal Series
  - Seasonal Factors
  - Seasonal Decomposition
Notation:

$D_t$: Demand observed in Period $t$.
$N$: Number of observations.
$F_t$: Forecast in Period $t-1$ made for Period $t$.
$e_t$: Forecast error in Period $t$ ($e_t = F_t - D_t$).

Moving Averages

$N$-Period Moving Average ($MA(N)$): Arithmetic average of the last $N$ observations.

$$F_t = \left(\frac{1}{N}\right) \left( D_{t-N} + D_{t-N+1} + \ldots + D_{t-2} + D_{t-1} \right)$$

Example 2 The data for the number of defective items that a manufacturer finds in the last eight batches of production are 100, 120, 88, 96, 112, 144, 152, 92. Considering that the batch size always remains the same, and, using the last eight-batch data, find the forecasts for the number of defective items in batches 9 & 10.
Exponential Smoothing

Forecast for Period $t$ is the weighted average of the last period’s observed demand & the forecast made for the last period in Period $t - 2$.

$$F_t = \alpha D_{t-1} + (1 - \alpha) F_{t-1}, \quad 0 < \alpha \leq 1$$

Recall that, $e_{t-1} = F_{t-1} - D_{t-1}$. Therefore,

$$F_t = \alpha D_{t-1} - \alpha F_{t-1} + F_{t-1} = -\alpha e_{t-1} + F_{t-1}$$

If $e_{t-1} < 0 \Rightarrow F_{t-1} < D_{t-1}$

$\Rightarrow$ Increase $F_{t-1}$ by a factor of $e_{t-1}$

If $e_{t-1} > 0 \Rightarrow F_{t-1} > D_{t-1}$

$\Rightarrow$ Decrease $F_{t-1}$ by a factor of $e_{t-1}$

<table>
<thead>
<tr>
<th>Batch</th>
<th># Defects</th>
<th>MA(8)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>100</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
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<td>100</td>
</tr>
<tr>
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<td>100</td>
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<td>5</td>
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<tr>
<td>6</td>
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<td>100</td>
</tr>
<tr>
<td>7</td>
<td>152</td>
<td>100</td>
</tr>
<tr>
<td>8</td>
<td>92</td>
<td>100</td>
</tr>
<tr>
<td>9</td>
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</tr>
</tbody>
</table>
Forecasting the Future is Always Difficult!

“Computers in the future may weigh no more than 1.5 tons.”
--Popular Mechanics, 1949

“I think there is a world market for maybe five computers.”
--Chairman of IBM, 1943.

“There is no reason anyone would want a computer in their home.”

“This telephone has too many shortcomings to be seriously considered as a means of communication. The device is inherently of no value to us.”
--Western Union internal memo, 1876.

“Airplanes are interesting toys but of no military value.”
--Professor of Strategy, Ecole Superieure de Guerre.

“Everything that can be invented has been invented.”